## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


## MATHEMATICS

0580/42
Paper 4 (Extended)

You must answer on the question paper.
You will need: Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For $\pi$, use either your calculator value or 3.142.


## INFORMATION

- The total mark for this paper is 130 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.

1 A grocer sells potatoes, mushrooms and carrots.
(a) A customer buys 3 kg of mushrooms at $\$ 1.04$ per kg and 4 kg of carrots at $\$ 1.28$ per kg . Calculate the total cost.

$$
\begin{equation*}
\$ \tag{2}
\end{equation*}
$$

(b) In one week, the ratio of the masses of vegetables sold by the grocer is

$$
\text { potatoes : mushrooms : carrots }=11: 8: 6 \text {. }
$$

(i) Work out the mass of mushrooms sold as a percentage of the total mass.
$\qquad$
(ii) The total mass of potatoes, mushrooms and carrots sold is 1500 kg .

Find the mass of carrots the grocer sells this week.
(iii) The profit the grocer makes selling 1 kg of carrots is $\$ 0.75$.

Find the total profit the grocer makes selling carrots this week.

## \$

(iv) On the last day of the week, the grocer reduces the price of 1 kg of potatoes by $8 \%$ to $\$ 1.15$. Calculate the original price of 1 kg of potatoes.

$$
\$
$$

(c) The grocer buys 620 kg of onions, correct to the nearest 20 kg . He packs them into bags each containing 5 kg of onions, correct to the nearest 1 kg .

Calculate the upper bound for the number of bags of onions that he packs.


NOT TO
SCALE
$A, B, C$ and $D$ are points on a circle.
$A D X$ and $B C X$ are straight lines.
Angle $B A D=x^{\circ}$ and angle $D C X=y^{\circ}$.
(a) Explain why $x=y$.

Give a geometrical reason for each statement you make.
(b) Show that triangle $A B X$ is similar to triangle $C D X$.
(c) $A D=15 \mathrm{~cm}, D X=9 \mathrm{~cm}$ and $C X=12 \mathrm{~cm}$.
(i) Find $B C$.

$$
\begin{equation*}
B C= \tag{3}
\end{equation*}
$$

$\qquad$
(ii) Complete the statement.

The ratio area of triangle $A B X$ : area of triangle $C D X=$ $\qquad$ : 1.

3 (a) The table shows information about the marks gained by each of 10 students in a test.

| Mark | 15 | 16 | 17 | 18 | 19 | 20 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 1 | 2 | 1 | 0 | 2 |

(i) Calculate the range.
(ii) Calculate the mean.
(iii) Find the median.
(iv) Write down the mode.
(b) Paulo's mean mark for 7 homework tasks is 17 .

After completing the 8th task, his mean mark is 17.5 .
Calculate Paulo's mark for the 8th task.
(c) The table shows the percentage scored by each of 100 students in their final exam.

| Percentage $(p)$ | $0<p \leqslant 30$ | $30<p \leqslant 50$ | $50<p \leqslant 60$ | $60<p \leqslant 70$ | $70<p \leqslant 100$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 12 | 18 | 35 | 20 | 15 |

On the grid, draw a histogram to show this information.


4 (a)


NOT TO
SCALE

The diagram shows a pyramid with a square base $B C D E$.
The diagonals $C E$ and $B D$ intersect at $M$, and the vertex $F$ is directly above $M$. $B E=12 \mathrm{~cm}$ and $F M=9 \mathrm{~cm}$.
(i) Calculate the volume of the pyramid.
[The volume, $V$, of a pyramid with base area $A$ and height $h$ is $V=\frac{1}{3} A h$.]
$\mathrm{cm}^{3}$
(ii) Calculate the total surface area of the pyramid.
(b)


NOT TO
SCALE

The diagram shows a toy made from a cone and a hemisphere.
The base radius of the cone and the radius of the hemisphere are both $r \mathrm{~cm}$. The slant height of the cone is $3 r \mathrm{~cm}$.

The total surface area of the toy is $304 \mathrm{~cm}^{2}$.
Calculate the value of $r$.
[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A=\pi r l$.] [The curved surface area, $A$, of a sphere with radius $r$ is $A=4 \pi r^{2}$.]

5 (a) (i) Factorise.

$$
x^{2}-x-12
$$

(ii) Simplify.

$$
\frac{x^{2}-16}{x^{2}-x-12}
$$

(b) Simplify.

$$
(2 x-3)^{2}-(x+1)^{2}
$$

(c) Write as a single fraction in its simplest form.

$$
\frac{2 x+4}{x+1}-\frac{x}{x-3}
$$

(d) Expand and simplify.

$$
(x-3)(x-5)(2 x+1)
$$

(e) Solve the simultaneous equations.

You must show all your working.

$$
\begin{aligned}
x-3 y & =13 \\
2 x^{2}-9 y & =116
\end{aligned}
$$

$$
\begin{array}{ll}
x=\ldots \ldots . . . . . . . . . . . . . . . ~ & y= \\
x=\ldots \ldots . . . . . . . . . . . . . . . ~ & y=
\end{array}
$$



The diagram shows triangle $A B C$ with $A B=17.2 \mathrm{~cm}$.
Angle $A B C=54^{\circ}$ and angle $A C B=68^{\circ}$.
(a) Calculate $A C$.

$$
A C=
$$

(b) $M$ lies on $B C$ and $M C=12.8 \mathrm{~cm}$.

Calculate $A M$.

$$
A M=
$$

(c) Calculate the shortest distance from $A$ to $B C$.
$7 \quad$ (a) $\quad \mathbf{p}=\binom{8}{-5} \quad \mathbf{q}=\binom{-4}{5}$
(i) Find $3 \mathbf{q}$.
(ii) (a) Find $\mathbf{p}-\mathbf{q}$.
(b) Find $|\mathbf{p}-\mathbf{q}|$.
(b)


In triangle $O M N, O$ is the origin, $\overrightarrow{O M}=\mathbf{a}$ and $\overrightarrow{O N}=\mathbf{b}$. $S$ is a point on $M N$ such that $M S: S N=5: 3$.

Find, in terms of $\mathbf{a}$ and/or $\mathbf{b}$, the position vector of $S$.
Give your answer in its simplest form.

8 (a) On the axes, sketch the graph of $y=4-3 x$.

(b) On the axes, sketch the graph of $y=-x^{2}$.

(c) (i) Find the coordinates of the turning points of the graph of $y=10+9 x^{2}-2 x^{3}$. You must show all your working.
$\qquad$
) and (
(ii) Determine whether each turning point is a maximum or a minimum.

Show how you decide.

9 (a) Janna and Kamal each invest $\$ 8000$.
At the end of 12 years, they each have $\$ 12800$.
(i) Janna invests in an account that pays simple interest at a rate of $r \%$ per year.

Calculate the value of $r$.

$$
\begin{equation*}
r= \tag{3}
\end{equation*}
$$

(ii) Kamal invests in an account that pays compound interest at a rate of $R \%$ per year. Calculate the value of $R$.

$$
R=
$$

(b) The population of a city is growing exponentially at a rate of $1.8 \%$ per year.

The population now is 260000 .
Find the number of complete years from now when the population will first be more than 300000 .

10 The table shows some values for $y=2 x^{3}+6 x^{2}-2.5$.

| $x$ | -3 | -2.5 | -2 | -1.5 | -1 | -0.5 | 0 | 0.5 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ |  | 3.75 | 5.5 | 4.25 | 1.5 |  | -2.5 | -0.75 |  |

(a) Complete the table.
(b) On the grid, draw the graph of $y=2 x^{3}+6 x^{2}-2.5$ for $-3 \leqslant x \leqslant 1$.

[4]
(c) By drawing a suitable line on the graph, solve the equation $2 x^{3}+6 x^{2}=4.5$.

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

$\qquad$ or $x=$ $\qquad$ or $x=$
(d) The equation $2 x^{3}+6 x^{2}-2.5=k$ has exactly two solutions.

Write down the two possible values of $k$.

$$
k=
$$

$\qquad$ or $k=$

11

$$
\mathrm{f}(x)=\frac{1}{x}, x \neq 0
$$

$g(x)=3 x-5$
$h(x)=2^{x}$
(a) Find.
(i) $\operatorname{gf}(2)$
(ii) $\mathrm{g}^{-1}(x)$

$$
\mathrm{g}^{-1}(x)=
$$

(b) Find in its simplest form $\mathrm{g}(x-2)$.
(c) Find the value of $x$ when
(i) $\mathrm{fg}(x)=0.1$

$$
x=
$$

(ii) $\mathrm{h}(x)-\mathrm{g}(7)=0$.

$$
\begin{equation*}
x= \tag{2}
\end{equation*}
$$

12 (a)


The diagram shows a circle of radius 12 cm , with a sector removed.
Calculate the perimeter of the remaining shaded shape.
cm [4]
(b) The diagram in part(a) shows the top of a cylindrical cake with a slice removed. The volume of cake that remains is $3510 \mathrm{~cm}^{3}$.

Calculate the height of the cake.

